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Cubic string field theory in pp-wave background and background independent Moyal structure

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Abstract: We study Witten open string field theory in the pp-wave background in the tensionless limit, and construct the N-string vertex in the basis which diagonalizes the string perturbative spectrum. We found that the Witten $*$-product can be viewed as infinite copies of the Moyal product with the same noncommutativity parameter $\theta = 2$. Moreover, we show that this Moyal structure is universal in the sense that, written in the string bit basis, Witten's $*$-product for any background can always be given in terms of the above-mentioned Moyal structure. We identify some projective operators in this algebra that we argue to correspond to D-branes of the theory.

Keywords: Bosonic Strings, String Field Theory, Penrose limit and pp-wave background.

Contents

1. Introduction

Recently, various aspects of string theory in pp-wave background are being studied with immense interests. The is partially due to the remarkable proposal [1] which states that a sector of the SYM operators with large R-charge is dual to the IIB string theory on a pp-wave background consisting of the metric

$$ds^2 = -\mu^2 x^i x^i (dx^+)^2 - 4dx^+ dx^- + \sum_{i=1}^{D-2} dx^i dx^i \,, \tag{1.1}$$

with $D-2=8$ transverse directions and a homogeneous RR 5-form flux.

The study of pp-wave background as an exact solution of string theory has a long history (see e.g. [2]–[5] and [6] for review and references therein). It was realised only recently that the pp-wave background is maximally supersymmetric [7] in the presence of a certain RR flux. Moreover the string model is exactly solvable [8, 9], just as in the NS-NS case. In this paper, we will concentrate on the simplest form of the pp-wave metric (1.1). The metric can be supported by different combinations of NS or RR flux and leads to different exactly solvable string models [8, 9, 10]. For example, in the maximally supersymmetric IIB pp-wave background [7], an RR flux $H_{+1234} = H_{+5678} = \mu$ is turned on. In the Nappi-Witten background (D=4) [11] and its higher dimensional generalization [12], the target space is the extended Heisenberg group H_n whose dimension is $D = 2n+2$;

and the background is supported by an NS flux $H_{+12} = \cdots = H_{+(2n+1)(2n+2)} = \mu$. These backgrounds are distinguished because they are exactly solvable. Here "exactly solvable" means that the model can be written in terms of free oscillators in the light cone gauge.

One of the main motivations in studying string field theory is the hope of a better understanding of the nonperturbative phenomena in string theory. Remarkable progress has been made recently in understanding D-branes and tachyon condensation [13] from the viewpoint of open string field theory [14]. In Witten's formulation, the open string $*$-product of fields Ψ_1, Ψ_2 is defined by identifying the right half of the string of Ψ_1 with the left half of the string of Ψ_2. The idea is made concrete [15] by using conformal theory techniques to provide a Fock space description of the Witten $*$-product. Recently, the work of Bars [16] has led to remarkable progress in identifying the Witten $*$-product as a continuous Moyal product. This new formulation may have the advantage in understanding the nonperturbative symmetries of string field theory based upon the progress of noncommutative field theory.

In this paper, we study the Witten open string field theory in the pp-wave background. In particular we would like to construct the Witten $*$-product as a first step to understand the algebraic structure of the string field theory. To avoid the complications due to picture changing operators in the supersymmetric case [17] (see for example [18] for more discussions), we will work in the bosonic case with the Nappi-Witten type background H_n. Since the string model is simplified and can be written in terms of free oscillators only in the light cone gauge, one can try to construct the covariant open string $*$-product in certain limiting situation. Inspired by the considerations in [19] (where the motivation was to consider specific limit, particularly the tensionless limit, such that the $AdS_5 \times S^5$ string model gets simplified and further understanding of the AdS/CFT correspondence can be achieved), we consider the tensionless limit of the bosonic string in the Nappi-Witten type pp-wave background. It has also been argued [20] that the tensionless limit should provide the proper starting point for investigating the high energy limit of string dynamics.

Witten's $*$-product is defined by the overlap conditions and is a background-independent concept. However the explicit representation of it, and hence the associated Moyal product, is basis dependent and background dependent. For the purpose of perturbative studies, it is convenient to use a basis which diagonalizes the hamiltonian. For example, in the flat background, the Witten $*$-product was constructed [15] in terms of the *harmonic oscillator basis* which diagonalizes the hamiltonian. In the tensionless limit, the string worldsheet is described most naturally in terms of an infinite number of string bits. In this *string bit basis*, the hamiltonian is diagonal. It is therefore natural to construct the N-string vertex in this basis. Using this vertex, we construct the corresponding Moyal structure following the ideas of [16]. We find that the Witten $*$-product is equivalent to a Moyal product and the Moyal structure consists of infinite copies of 2-dimensional planes with the same noncommutativity parameter $\theta = 2$. This is one of the main result of this paper. This construction using the string bit basis leads us to the observation that the equivalence of the Witten $*$-product with the above-mentioned Moyal product is indeed *background independent*. This is another main result of this paper.

The paper is organized as follows. In section 2, we perform the tensionless limit of the bosonic open string in the Nappi-Witten type pp-wave background. In section 3, we construct the N-string vertex in the string bit basis by solving the overlap conditions. It turns out that the Witten $*$-product is equivalent to infinite copies of the Moyal product with the same noncommutativity $\theta = 2$, together with the midpoint coordinates which are commutative. In section 5 we point out that the Witten $*$-product for flat background approaches to the same $\theta = 2$ algebra in the high energy limit. In fact, in the string bit basis, one can always identify Witten's product with this algebra. The open string field theory action is constructed in section 6. Projective operators corresponding to D-branes are discussed in section 7. Finally we conclude in section 8.

2. Tensionless string in pp-wave background

The Nappi-Witten type string model is a WZW model based on the extended Heisenberg group H_n [12]. The target space H_n is of dimension $D = 2n + 2$ and has the metric (1.1). There is also an NS flux

$$H_{+(2i+1)(2i+2)} = \mu\,, \qquad i = 0, \dots, n\,. \tag{2.1}$$

The string action in the conformal gauge takes the form

$$\begin{aligned} S' = \frac{1}{4\pi\alpha'}\int d\tau \int_0^\pi d\sigma \Big[& \dot{X}^2 - X'^2 - \mu^2 X^2 \left(\dot{X}^{+2} - X'^{+2}\right) - 4(\dot{X}^+\dot{X}^- - X'^+X'^-) + \\ & + iB_{\mu\nu}\left(\dot{X}^\mu X'^\nu - \dot{X}^\nu X'^\mu\right)\Big]\,, \end{aligned} \tag{2.2}$$

where the "level" k has been identified with α'^{-1}. To take the tensionless limit

$$\alpha' \to \infty\,, \tag{2.3}$$

we rescale the time coordinate

$$t = \alpha'\tau\,. \tag{2.4}$$

Noticing that $B_{\mu\nu}$ is independent of α', we obtain

$$S = \frac{1}{4\pi}\int dt \int_0^\pi d\sigma \left((\partial_t X)^2 - (\mu\partial_t X^+)^2 X^{i2} - 4\partial_t X^+ \partial_t X^-\right) \tag{2.5}$$

in the limit $\alpha' \to \infty$. As it can be seen clearly from this action, the string is essentially a combination of infinitely many independent points labelled by $\sigma \in [0, \pi]$. Also note that a new symmetry corresponding to arbitrary permutation of σ emerges in this limit.

According to (2.5), the hamiltonian is

$$H = \pi \int_0^\pi d\sigma \left(P^{i\,2} + \left(\frac{\mu P_-}{2}\right)^2 X^{i\,2} - P_+P_-\right), \tag{2.6}$$

where the conjugate momenta are

$$P_- = \frac{-1}{\pi}\partial_t X^+\,, \qquad P_+ = \frac{-1}{\pi}\left(\partial_t X^- + \frac{1}{2}\mu^2 X^{i\,2}\partial_t X^+\right), \qquad P^i = \frac{1}{2\pi}\partial_t X^i\,. \tag{2.7}$$

The equations of motion are

$$\partial_t^2 X^i + (\pi\mu P_-)^2 X^i = 0\,, \tag{2.8}$$

$$\partial_t P_+ = 0\,, \qquad \partial_t P_- = 0\,. \tag{2.9}$$

Due to translational invariance in X^+, X^-, the light-cone momenta P_+, P_- are conserved. It is clear that each transverse coordinate $X^i(\sigma)$ at a given point σ on the string corresponds to a simple harmonic oscillator

$$a^i(\sigma) := \sqrt{\frac{\pi}{\omega_0(\sigma)}}\left(P^i(\sigma) - i\frac{\omega_0(\sigma)}{2\pi}X^i(\sigma)\right) \tag{2.10}$$

with angular frequency

$$\omega_0(\sigma) = \pi\mu|P_-(\sigma)|\,. \tag{2.11}$$

The commutation relations of $a^i(\sigma)$ are

$$[a^i(\sigma), a^j(\sigma')^\dagger] = \delta^{ij}\delta(\sigma-\sigma')\,. \tag{2.12}$$

The part of the hamiltonian (2.6) relevant to X^i is

$$H_X = \int_0^\pi d\sigma\ \omega_0(\sigma)\sum_i a^{i\dagger}(\sigma)a^i(\sigma) \tag{2.13}$$

up to normal ordering. The eigenstates of H are spanned by

$$\bigotimes_{\sigma\in[0,\pi]} |n^i(\sigma); P_+(\sigma), P_-(\sigma)\rangle\,, \tag{2.14}$$

where n^i is the simple harmonic oscillator label for X^i.

In the next section, we will construct the Witten $*$-product in terms of this pointwise basis of oscillators. Note that in the tensionless limit, the string becomes ultra-local and there is no need to impose any boundary condition on the open string described by the action (2.5). The pointwise basis is the appropriate basis for describing the string configurations in the tensionless limit. It would be insufficient to use the usual oscillator basis a_n which is defined in terms of a Fourier basis $\cos n\sigma$, since Neumann boundary condition is implied if one use this basis. Moreover, as we will show, the pp-wave hamiltonian is diagonalized in this basis, it is therefore also an useful choice. Most importantly, we will show in section 5 that written in this basis, Witten's $*$-product has a simple universal representation in terms of the Moyal product, independent of the string background. Hopefully this will provide some insights into the algebraic structures of the open string field theory at finite α'.

We remark that to recover the usual string picture with a finite α', we expect that a nontrivial perturbation $\mathcal{O}$ can be introduced to the tensionless theory

$$S = S_0 + \lambda\mathcal{O}\,, \qquad \lambda \sim l_s^{-1} \tag{2.15}$$

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that implement the effects of having a nonzero tension. The perturbation $\mathcal{O}$ should introduce correlation for neighbouring string bits and implement the Neumann boundary condition. In this paper, we will be contended with the zeroth order construction.

It is also useful to use a string-bit regularization by discretizing the worldsheet into a large number J of bits and write

$$a(\sigma) = \sqrt{\frac{J}{\pi}} \sum_{\mathrm{c}=0}^{J-1} a_{\mathrm{c}} \chi_{\mathrm{c}}(\sigma) \,, \tag{2.16}$$

where $\chi_{\mathrm{c}}(\sigma)$ is a step function with support on the interval $[\pi \mathrm{c}/J, \pi(\mathrm{c}+1)/J]$. In this regularization,

$$\int d\sigma \to \frac{\pi}{J} \sum_{\mathrm{c}} \,, \tag{2.17}$$

and string excitations in the transverse directions are characterized by the oscillators a_c^i obeying

$$[a_{\mathrm{c}}^i, a_{\mathrm{d}}^{j\dagger}] = \delta^{ij} \delta_{\mathrm{cd}} \,, \qquad \mathrm{c}, \mathrm{d} = 0, \ldots, J-1 \,. \tag{2.18}$$

We will present our results in the following in terms of $a^i(\sigma)$, with the understanding that the discrete basis is employed whenever it is necessary, e.g. (3.12).

Finally we remark that the usefulness of the pointwise basis $a(\sigma)$ was also pointed out in [21], where the N-string vertex were constructed, and it was argued to be a convenient choice since it gives additional insights into the algebraic structure of the open string field theory.

3. N-string vertex

Up to an overall normalization, the N-string vertex is determined by the overlap conditions,

$$\left(X_I^\mu(\sigma) - X_{I-1}^\mu(\pi-\sigma)\right) |V_N\rangle = 0 \,, \tag{3.1}$$

$$\left(P_I^\mu(\sigma) + P_{I-1}^\mu(\pi-\sigma)\right) |V_N\rangle = 0 \,, \qquad \sigma \in \left[0, \frac{\pi}{2}\right] , \tag{3.2}$$

where $I \in \mathbf{Z}_N$ is the label for different strings, and X_I and P_I are the string coordinates and conjugate momenta for the I-th string.

We now assume that the full solution to (3.1) and (3.2) is of the form

$$|V_N\rangle = |V_N\rangle_\pm |V_N\rangle_X \,. \tag{3.3}$$

It is easy to see that the light cone part $|V_N\rangle_\pm$ is given by

$$|V_N\rangle_\pm = \prod_{I=1}^{N} \prod_{\sigma\in[0,\pi/2]} \delta(\vec{P}_I(\sigma) + \vec{P}_{(I-1)}(\pi-\sigma)) |\vec{P}_1\rangle \otimes \cdots \otimes |\vec{P}_N\rangle \,, \tag{3.4}$$

where $|\vec{P}_I\rangle$ is a (light cone) momentum eigenstate with eigenvalue $\vec{P}_I$ for the I-th string. We recall that the hamiltonian (2.6) is diagonal in this basis $|\vec{P}_I\rangle$.

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We will skip the index i for transverse directions from now on. The overlap condition (3.1) and (3.2) implies that

$$\frac{1}{\sqrt{\mu|P_{-I}(\sigma)|}}|V_N\rangle_\pm \cdot \left(\tilde{X}_I(\sigma)-\tilde{X}_{I-1}(\pi-\sigma)\right)|V_N\rangle_X = 0\,, \tag{3.5}$$

$$\sqrt{\mu|P_{-I}(\sigma)|}|V_N\rangle_\pm \cdot \left(\tilde{P}_I(\sigma)-\tilde{P}_{I-1}(\pi-\sigma)\right)|V_N\rangle_X = 0\,, \tag{3.6}$$

where

$$\tilde{X}_I(\sigma) := i\left(a_I(\sigma)-a_I^\dagger(\sigma)\right), \qquad \tilde{P}_I(\sigma) := \frac{1}{2}\left(a_I(\sigma)+a_I^\dagger(\sigma)\right). \tag{3.7}$$

The overlap condition for the transverse part $|V_N\rangle_X$ is thus

$$\left(\tilde{X}_I(\sigma)-\tilde{X}_{I-1}(\pi-\sigma)\right)|V_N\rangle_X = 0\,, \tag{3.8}$$

$$\left(\tilde{P}_I(\sigma)-\tilde{P}_{I-1}(\pi-\sigma)\right)|V_N\rangle_X = 0\,, \qquad \sigma\in\left[0,\frac{\pi}{2}\right]. \tag{3.9}$$

The overlap condition relates a point σ on the string to the point $(\pi-\sigma)$ on another string. The only fixed point of the map $\sigma\to(\pi-\sigma)$ is the midpoint $\sigma=\pi/2$, and thus the midpoint should be dealt with separately. The vertex should be decomposable as

$$|V_N\rangle_X = |V_N^0\rangle_X|V_N'\rangle_X\,, \tag{3.10}$$

where $|V_N^0\rangle_X$ and $|V_N'\rangle_X$ are the vertices for the midpoint and the rest of the string, respectively. The overlap condition for the midpoint is simply to identify $\tilde{X}_I^i(\pi/2)$ for all I. Thus it is easy to find

$$\begin{aligned}|V_N^0\rangle_X &= \int dx|x\rangle\otimes\cdots\otimes|x\rangle \\ &= \int\prod_{I=1}^N\frac{dp_I}{2\pi}\delta\left(\sum_{J=1}^N p_J\right)|p_1\rangle\otimes\cdots\otimes|p_N\rangle\,,\end{aligned} \tag{3.11}$$

where $|x\rangle$ and $|p\rangle$ are the eigenstates of $\tilde{X}(\pi/2)$ and $\tilde{P}(\pi/2)$. The overlap conditions for $\sigma\neq\pi/2$ can be solved by

$$|V_N'\rangle_X = \exp\left[-\sum_{i=1}^N\int_{0\le\sigma<\pi/2}d\sigma a_I^\dagger(\sigma)a_{I-1}^\dagger(\pi-\sigma)\right]|0\rangle_X\,, \tag{3.12}$$

where the vacuum $|0\rangle_X$ is defined to be

$$a_I(\sigma)|0\rangle_X = 0\,, \qquad \text{for } 0\le\sigma\le\pi \quad\text{and}\quad \sigma\neq\frac{\pi}{2}. \tag{3.13}$$

Here the formula (3.12) is interpreted in terms of the string bit picture (2.16)-(2.18) since it is crucial to treat the midpoint separately. We note that, when acting on $|V_N\rangle_X$, it is

$$a_I(\sigma)\simeq -a_{I-1}^\dagger(\pi-\sigma)\,, \qquad a_{I-1}(\pi-\sigma)\simeq -a_I^\dagger(\sigma)\,, \qquad \text{for } \sigma\in\left[0,\frac{\pi}{2}\right). \tag{3.14}$$

For readers who prefer expansion of open string modes in the oscillation basis, we comment that a more careful treatment on the basis transfer can be found in [16, 22].

4. Witten $*$ as Moyal $*$

In this section, we will first review how the Moyal $*$-product is obtained from the Witten $*$-product in section 4.1. In [16, 23], this relation is performed using the harmonic oscillator basis. Then in section 4.2 we will show that a slight variation of the method can be adapted for the string bit basis, which is the most natural and convenient basis in the tensionless limit. A Moyal structure with infinite copies of noncommutative planes with $\theta = 2$ is obtained.

4.1 The stereotype

Let us start with the flat case and with a single pair of string oscillators. To interpret Witten's $*$-product as the Moyal product with noncommutativity θ we recall that one need to put $|V_3\rangle$ in the standard form [23]

$$\begin{aligned}|V_3\rangle = \frac{2}{3\sqrt{\pi}}\frac{1}{1+\frac{\theta^2}{12}}\exp\Bigg\{\sum_{I=1,2,3}\Bigg[&-\frac{1}{2}\left(\frac{-4+\theta^2}{12+\theta^2}\right)(e_I^\dagger e_I^\dagger + o_I^\dagger o_I^\dagger) - \\ &-\left(\frac{8}{12+\theta^2}\right)(e_I^\dagger e_{I+1}^\dagger + o_I^\dagger o_{I+1}^\dagger) - \\ &-\left(\frac{4i\theta}{12+\theta^2}\right)(e_I^\dagger o_{I+1}^\dagger - o_I^\dagger e_{I+1}^\dagger)\Bigg]\Bigg\}|0\rangle\,, \end{aligned} \tag{4.1}$$

for some creation and annihilation operators $(e, e^\dagger, o, o^\dagger)$ satisfying

$$[o, o^\dagger] = [e, e^\dagger] = 1\,, \qquad [o, e^\dagger] = [e, o^\dagger] = 0\,. \tag{4.2}$$

The associated Moyal product is defined on a two dimensional plane (y, z), where (y, z) label the spectrum of the position operators $\hat{y}, \hat{z}$ defined by ($[\hat{y}, \hat{z}] = 0$),

$$\hat{y} := \frac{i}{\sqrt{2}}(e - e^\dagger)\,, \qquad \hat{z} := \frac{i}{\sqrt{2}}(o - o^\dagger)\,. \tag{4.3}$$

The corresponding eigenstate is

$$\langle y, z| = \frac{1}{\sqrt{\pi}}\langle 0|\exp\left(-\frac{1}{2}(y^2+z^2) + i\sqrt{2}(ey+oz) + \frac{1}{2}(ee+oo)\right)\,. \tag{4.4}$$

The coordinates y, z satisfy the $*$ commutation relation

$$[y, z]_* = i\theta\,, \tag{4.5}$$

where the Moyal $*$-product is defined through $|V_3\rangle$ according to the correspondence

$$(\langle f_1| \otimes \langle f_2| \otimes \langle y_3, z_3|)|V_3\rangle = (f_1 * f_2)(y_3, z_3)\,, \tag{4.6}$$

where

$$\langle f| = \int dydz f(y,z)\langle y, z|\,. \tag{4.7}$$

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The noncommutativity parameter θ is directly related to the spectrum of the Neumann matrices that appears in the 3-strings vertex.

The above consideration for a single pair of string oscillators (e,o) can be easily generalized [23] to the full string vertex. In the flat case, $|V_3\rangle$ in the oscillator basis takes the form

$$|V_3\rangle = \exp\left(-\frac{1}{2}\sum_{I=1}^{3} a_I^\dagger U^{IJ} a_J^\dagger\right)|0\rangle\,, \tag{4.8}$$

and the following constraints are satisfied

$$\begin{aligned} U^{IJ} &= U^{(I+1)(J+1)}\,, && (4.9)\\ M^{12}+M^{21} &= 1-M^{11}\,, && (4.10)\\ M^{12}M^{21} &= M^{11}(M^{11}-1)\,, && (4.11)\end{aligned}$$

where

$$M^{11} = CU^{11}\,, \qquad M^{12} = CU^{12}\,, \qquad M^{21} = CU^{21} \tag{4.12}$$

and C is the matrix

$$C_{mn} = (-1)^n\delta_{mn}\,. \tag{4.13}$$

It was shown that the Witten $*$-product can be interpreted as a Moyal $*$-product in terms of suitable variables [16, 23] The noncommutativity parameter θ is related to the eigenvalues λ^{IJ} of the Neumann matrices M^{IJ} by

$$\lambda^{11} = \frac{-4+\theta^2}{12+\theta^2}\,, \qquad \lambda^{12}+\lambda^{21} = \frac{16}{12+\theta^2}\,, \qquad \lambda^{12}-\lambda^{21} = \frac{8\theta}{12+\theta^2}\,. \tag{4.14}$$

In general, if the 3-string vertex satisfies the properties (4.9)–(4.11), we can hope that Witten's product can be written as a Moyal product.

4.2 Moyal product for pp-wave

To put $|V_3\rangle_X$ of (3.12) in the standard form (4.1), we define a new basis of oscillators $\{e(\sigma), o(\sigma)\}$ by

$$a^\dagger(\sigma) = \frac{1}{\sqrt{2}}(e^\dagger(\sigma)+io^\dagger(\sigma))\,, \qquad a^\dagger(\pi-\sigma) = \frac{1}{\sqrt{2}}(e^\dagger(\sigma)-io^\dagger(\sigma))\,, \qquad \sigma\in\left[0,\frac{\pi}{2}\right)\,. \tag{4.15}$$

The oscillators e, o satisfy the commutation relations

$$[o(\sigma), o^\dagger(\sigma')] = [e(\sigma), e^\dagger(\sigma')] = \delta(\sigma-\sigma')\,, \qquad [o(\sigma), e^\dagger(\sigma')] = [e(\sigma), o^\dagger(\sigma')] = 0\,. \tag{4.16}$$

Then one introduces the infinite copies of 2-dimensional planes whose coordinates $y(\sigma), z(\sigma)$ are the eigenvalues of the position operators ($[\hat{y}(\sigma), \hat{z}(\sigma')] = 0$),

$$\hat{y}(\sigma) = \frac{i}{\sqrt{2}}(e(\sigma)-e^\dagger(\sigma)) = \frac{1}{2}(\tilde{X}(\sigma)+\tilde{X}(\pi-\sigma))\,, \tag{4.17}$$

$$\hat{z}(\sigma) = \frac{i}{\sqrt{2}}(o(\sigma)-o^\dagger(\sigma)) = -\tilde{P}(\sigma)+\tilde{P}(\pi-\sigma)\,, \qquad \text{for } \sigma\in\left[0,\frac{\pi}{2}\right)\,. \tag{4.18}$$

Then one sees that the Witten $*$-product defined by $|V_3\rangle_X$ is equivalent to the Moyal $*$-product

$$[y(\sigma), z(\sigma')]_* = 2i\delta(\sigma - \sigma'), \qquad \sigma, \sigma' \in \left[0, \frac{\pi}{2}\right) \tag{4.19}$$

with the same noncommutativity parameter $\theta = 2$. Note that the algebra (4.19) is not defined for $\sigma = \pi/2$ because $z(\pi/2) = 0$, that is, the midpoint $y(\pi/2) = \sqrt{2}X(\pi/2)$ is a single commutative coordinate. This is natural because there is no distinction between left and right at the midpoint. We shall denote the midpoint coordinate by $x \equiv y(\pi/2)$.

To display the Moyal product more explicitly, let us define the string wave function $\Psi(x, y, z, P_+, P_-)$ using the basis

$$\langle x, y, z, \vec{P}| := \langle x|\langle y, z|\langle P_+, P_-|\,. \tag{4.20}$$

That is,

$$\langle\Psi| = \int dx Dy Dz D\vec{P} \;\; \Psi(x, y, z, \vec{P})\langle x, y, z, \vec{P}|\,, \tag{4.21}$$

where

$$D\vec{P} = \prod_{\sigma\in[0,\pi]} dP_+(\sigma)dP_-(\sigma)\,. \tag{4.22}$$

The full Moyal $*$-product is defined through the Witten $*$-product by

$$(\Psi_1 *_M \Psi_2)(x, y, z, \vec{P}) = \left(\langle\Psi_1| \otimes \langle\Psi_2| \otimes \langle x, y, z, \vec{P}|\right) |V_3\rangle\,. \tag{4.23}$$

It turns out to be

$$\begin{aligned}(f *_M g)(x, y, z, \vec{P}) = \int D\vec{P}_1 D\vec{P}_2 \; & f(x, y, z, \vec{P}_1) * g(x, y, z, \vec{P}_2)\delta(\vec{P}(\sigma) + \vec{P}_2(\pi - \sigma)) \times \\ & \times \delta(\vec{P}_2(\sigma) + \vec{P}_1(\pi - \sigma))\delta(\vec{P}_1(\sigma) + \vec{P}(\pi - \sigma))\end{aligned} \tag{4.24}$$

where $*$ for the 2-dimensional planes is defined by (4.19):

$$e^{i\int_{0\leq\sigma<\pi/2} d\sigma(\partial_{y_1}\partial_{z_2} - \partial_{z_1}\partial_{y_2})}|_{y_1=y_2=y,\, z_1=z_2=z}\,. \tag{4.25}$$

We note that the noncommutative algebra (4.19) is the same as the large κ limit of the flat background case [16, 23] (see also (5.1) below). This is not surprising because the tensionless limit is a high energy limit. We will have more comments on this in the next section. We also note that this algebra possesses the permutation symmetry S_∞ that allows one to exchange any two points σ and σ' on the string for $\sigma, \sigma' < \pi/2$, with their mirror images (with respect to the midpoint of the string) $(\pi - \sigma)$ and $(\pi - \sigma')$ also swapped at the same time. For this to be the symmetry of the string field theory, we need to check that the BRST operator is invariant. The BRST operator in Segal's gauge is constructed below from (6.2), and this is indeed the case.

5. Background independence of star product

In this section, we will show that the Witten $*$-product, when expressed in terms of the string bit basis, can always be written in terms of the universal Moyal structure with $\theta = 2$.

How is the algebra (4.24) related to the algebra for the flat background? The Neumann matrices are diagonalized for the flat background in [24] (apart from the zero mode) and the corresponding Moyal product has [23]

$$\theta = 2\tanh\left(\pi\frac{\kappa}{4}\right), \tag{5.1}$$

where $\kappa \in [0, \infty)$ is the index for eigenvectors. The eigenvectors $v_n(\kappa)$ for the flat background can be obtained from the generating function

$$f_\kappa(z) = \sum_{n=1}^{\infty} \frac{v_n(\kappa)}{\sqrt{n}} z^n \propto (1 - \exp(-\kappa \tan^{-1} z)) \,. \tag{5.2}$$

When we increase the parameter κ, the eigenvector $v(\kappa)$ tends to have more contribution from larger oscillation modes. The peak of $v_n(\kappa)$ is at around $n \sim \log \kappa$ for large κ. As a result, roughly speaking, in the high energy limit we should consider modes with large κ, and for $\kappa \to \infty$, the parameter θ has a finite limit at $\theta = 2$. Therefore we find that the Witten product corresponds to the Moyal product with $\theta = 2$ for both pp-wave and flat backgrounds in the high energy limit. This suggests that the Witten $*$-product can always be written in terms of this universal Moyal structure for any background in the tensionless limit. As we shall see, it is even more general than this.

It is useful to recall that the great simplification in using the string bit basis is that it provides a very clear exposition of the algebraic structure of the Witten product. Moreover it also diagonalizes the hamiltonian in the tensionless limit. Therefore it is not just natural, but also convenient to use this basis to construct the string vertices and to represent the Witten product in the high energy limit. When α' is finite, the string bit basis no longer diagonalizes the hamiltonian. However its algebraic advantages remain. In fact, for any background, the Witten $*$-product written in the string bit basis can always be presented in terms of the Moyal product with the universal noncommutativity parameter $\theta = 2$.

It is easy to see this. Note that for any background, one can simply use the string bit basis of creation and annihilation operators $a(\sigma), a^\dagger(\sigma)$ defined by

$$X(\sigma) = i(a(\sigma) - a^\dagger(\sigma)) \,, \qquad P(\sigma) = \frac{1}{2}(a(\sigma) + a^\dagger(\sigma)) \,, \tag{5.3}$$

and define the state $|0\rangle$ by $a(\sigma)|0\rangle = 0$. Immediately we find that the N-string vertices given by the same expressions (3.10), (3.11) and (3.12) for $|V_N\rangle$ satisfy all the overlap equations. We stress that this holds for any α'.

In the literature, the N-string vertices are often constructed in terms of the perturbative vacuum and the creation, annihilation operators for the perturbative spectrum, which are useful for a perturbative description. However, as a nonperturbative formulation, the string field theory is background independent. It is not always necessary to write down everything in accordance with the perturbative spectrum. What we just showed is that, independent of the string background, one can always represent the Witten product as the Moyal product with $\theta = 2$ in the string bit basis; and that in the tensionless limit, this basis also diagonalizes the hamiltonian and so should be a useful one. We expect that this background independent representation of the Witten $*$-product as a universal Moyal product should provide insights into a better understanding of the formal aspects of Witten's open string field theory.

Before we move on, we remark that in a generic background, the permutation symmetry S_∞ is preserved only in the high energy (tensionless) limit and is always broken in the low energy by the kinetic term. It will be interesting to see if there is any background in which this global symmetry is unbroken and the Moyal product takes the simple form even for finite α'.

6. String field theory action

We focus on the matter part of the string field theory action, which has the following kinetic term in the Siegel gauge ($b_0\Psi = 0$)

$$\frac{1}{2\alpha'}\int \Psi(L_0 - 1)\Psi\,. \tag{6.1}$$

In the pp-wave background with $\alpha' \to \infty$, $L_0 = \alpha' H$ (2.6) is the Virasoro operator with respect to τ

$$L_0 = \pi\alpha' \int_0^\pi d\sigma \left(P^2 + \left(\frac{\mu P_-}{2}\right)^2 X^2 - P_+ P_-\right). \tag{6.2}$$

One can rewrite everything in terms of $x, y, z, \vec{P}$ by defining the derivatives of $y(\sigma), z(\sigma)$

$$\frac{\partial}{\partial y(\sigma)} = i(\tilde{P}(\sigma) + \tilde{P}(\pi - \sigma))\,, \qquad \frac{\partial}{\partial z(\sigma)} = \frac{i}{2}(\tilde{X}(\sigma) - \tilde{X}(\pi - \sigma)) \tag{6.3}$$

and use linear combinations of y, z and their derivatives to express P and X in (6.2). In the large α' limit we can replace $(L_0 - 1)$ by L_0 for the kinetic term, and the factor of α' in L_0 will cancel the α' in the denominator of the kinetic term in (6.4).

Including the cubic term, the full string field theory action in the tensionless limit is

$$S = \frac{-1}{g^2}\left(\frac{1}{2}\int \Psi H \Psi + \frac{1}{3}\int \Psi *_M \Psi *_M \Psi\right), \tag{6.4}$$

where g is the open string coupling, and

$$H = \pi \int_0^{\pi/2} d\sigma \left[-\frac{1}{2}\mu|P_-(\sigma)|(\partial_y^2 + \partial_z^2 - y^2 - z^2) + P_+(\sigma)P_-(\sigma) + P_+(\pi - \sigma)P_-(\pi - \sigma)\right]. \tag{6.5}$$

7. Projective operators and D-branes

For the flat background, it was conjectured that D-branes are projective states [24] in the vacuum string field theory (VSFT) [13]. It has been checked that, in addition to giving the correct ratio of brane tensions [24, 25], the tachyon state of open strings ending on D-branes are reproduced [26, 27, 28] from the VSFT. In this section we will follow the assumptions and formulations of the vacuum string field theory (VSFT) [13] in flat space and consider projections as candidates of D-branes in the pp-wave background for the tensionless limit.

The projections are usually constructed in the oscillator basis, but we will see that it is easier to consider them as projective functions of the Moyal variables. Due to the simplicity of our Moyal product, our job here is much simpler than the flat case.

Since the midpoint is a commutative coordinate as noted before, it is more convenient to separate it from the others and define the variables

$$Y(\sigma) = y(\sigma) - \frac{1}{\sqrt{2P_-(\pi/2)}}\left(\sqrt{P_-(\sigma)} + \sqrt{P_-(\pi-\sigma)}\right) x\,, \tag{7.1}$$

$$Z(\sigma) = z(\sigma)\,, \tag{7.2}$$

$$x = \sqrt{\mu P_-(\pi/2)}X\left(\frac{\pi}{2}\right), \tag{7.3}$$

so that

$$[Y(\sigma), Z(\sigma)]_* = 2i\delta(\sigma-\sigma')\,, \qquad [x, Y(\sigma)]_* = [x, Z(\sigma)]_* = 0\,, \tag{7.4}$$

and more importantly, $Y(\sigma), Z(\sigma)$ are invariant under a shift of $X(\sigma)$. A string state Ψ can be viewed as a function of $Y(\sigma), Z(\sigma), \vec{P}(\sigma)$ and the midpoint coordinate x. Based on the relation (7.4), we consider projective states which take the factorized form:

$$\Psi(x, Y, Z, \vec{P}) = \Phi(\vec{P})\Phi'(Y, Z)\Phi''(x)\,, \tag{7.5}$$

where each factor is a projective function.

First we consider the factor $\Phi(\vec{P})$, which can be further decomposed as

$$\Phi = h\cdot g\,, \tag{7.6}$$

where $h(\vec{P}(\pi/2))$ depends only on $\vec{P}(\pi/2)$ and $g(\vec{P})$ depends on $\vec{P}(\sigma)$ for $\sigma \neq \pi/2$. For g to be a projection, we need

$$\begin{aligned} g(\vec{P}_3) = &\int D\vec{P}_1 D\vec{P}_2\; g(\vec{P}_1)g(\vec{P}_2)\delta(\vec{P}_1(\sigma) + P_3(\pi-\sigma)) \times \\ &\times \delta(\vec{P}_2(\sigma) + \vec{P}_1(\pi-\sigma))\delta(\vec{P}_3(\sigma) + \vec{P}_2(\pi-\sigma))\,. \end{aligned} \tag{7.7}$$

The simplest possibility is for g to be a constant, independent of $\vec{P}$. A few less trivial cases are:

$$g(\vec{P}) \propto \delta\left(\vec{P}\left(\frac{\pi}{2}\right)\right)\prod_{\sigma<\pi/2}\delta(\vec{P}(\sigma) + \vec{P}(\pi-\sigma))\,, \tag{7.8}$$

$$g(\vec{P}) \propto \delta(\vec{P}_L)\delta(\vec{P}_R)\,, \tag{7.9}$$

$$g(\vec{P}) \propto \delta(P_+(\sigma) - P_-(\sigma))\,. \tag{7.10}$$

The condition for h analogous to (7.7) is

$$h(\vec{P}_3) = \int d\vec{P}_1 d\vec{P}_2 h(\vec{P}_1) h(\vec{P}_2) \delta(\vec{P}_1 + \vec{P}_2) \delta(\vec{P}_2 + \vec{P}_3) \delta(\vec{P}_3 + \vec{P}_1) \,. \tag{7.11}$$

It follows that

$$h\left(\vec{P}\left(\frac{\pi}{2}\right)\right) \propto \delta\left(\vec{P}\left(\frac{\pi}{2}\right)\right). \tag{7.12}$$

This implies that the our assumptions about factorization (7.5) works only for lumps stretched in the X^- direction.

For $\Phi'(Y,Z)$, the simplest choice is to take it to be the noncommutative GMS solitons [29]. For example, $\Phi'(Y,Z)$ can be taken as

$$\mathcal{N} e^{-\frac{1}{2}\int d\sigma (Y^2(\sigma) + Z^2(\sigma))} \equiv |0\rangle\!\rangle \langle\!\langle 0| \,, \tag{7.13}$$

where $\mathcal{N}$ is a normalization constant depending on spectral measure. In the discrete string bit basis (2.16), $\mathcal{N} = 2^{\frac{J}{\pi}}$. In the flat case, $|0\rangle\!\rangle\langle\!\langle 0|$ corresponds to the ground state $|0\rangle$ of the perturbative string. This is consistent with the result obtained in the oscillator basis [24]. There, $T = M = 0$ for $\theta = 2$, so that the sliver state is proportional to $|0\rangle$. All rank-one projector states such as $|n\rangle\!\rangle\langle\!\langle n|$ can be obtained from (7.13) by unitary transformations, and be mapped into the oscillator basis with respect to $|V_3\rangle$. These states are new projectors of vacuum string field theory and are different from the sliver state, more details on this point for the flat background can be found in [30].

Finally we consider $\Phi''(x)$. Any state for the midpoint coordinate can be written in the form

$$\int dx f(x) |x\rangle \,. \tag{7.14}$$

It is a projection if $f^2(x) = f(x)$. A projection for a lump extending from x_1 to x_2 is of the form

$$\Phi''(x) = \Theta(x - x_1) - \Theta(x - x_2) \,, \tag{7.15}$$

for $x_1 < x_2$. $\Theta(x)$ is the step function which equals one or zero depending on whether x is positive or negative.

At this point it is not clear whether any of these projective states considered above represents D-branes. A nontrivial test for the sliver state to be a D-brane in the context of vacuum string field theory in flat background [24] is the decent relation of the D-brane tension, that is

$$\frac{T_p}{2\pi\sqrt{\alpha'}\,T_{p+1}} = 1 \,. \tag{7.16}$$

Another more difficult test is to look for open string states in the background of D-branes [26, 27, 28]. The latter requires a lot more work, and we leave it for the future.

Let us first review the sliver states in the flat background. In the flat case, the longitudinal direction of a D-brane is constructed with respect to the zero momentum sector of the 3-string vertex in the momentum basis [24], denoted as $|V_3\rangle_{p_0=0}$. The D-brane's longitudinal directions are thus infinite due to the fixed zero momentum. On the other hand, the transverse direction is constructed with respect to the 3-string vertex in the oscillator basis, which is denoted as $|V_3\rangle_o$, and is related to $|V_3\rangle_{p_0}$ by

$$x_0 = \frac{i}{2\sqrt{b}}(a_0 - a_0^\dagger)\,, \qquad p_0 = \sqrt{b}(a_0 + a_0^\dagger)\,, \tag{7.17}$$

where b represents the thickness of the D-brane which breaks translational invariance. Although the 3-string vertex in both bases are mathematically equivalent, the corresponding sliver states are not. An important result [24] is that the ratio (7.16) is independent of b.

In the Moyal basis of [23], $|V_3\rangle_{p_0=0}$ and $|V_3\rangle_o$ are mathematically of the same form (4.1) in the continuous spectrum[1] [31]. The difference between the sliver states for Dp-brane and D$(p+1)$-brane corresponds to different choices of the representation for the commutative variables with $\theta(\kappa = 0) = 0$. The one for $|V_3\rangle_{p_0=0}$ is $P_L - P_R$ (or equivalently P_L, since $p_0 = 0$) and the one for $|V_3\rangle_o$ is the midpoint coordinate $X(\pi/2)$. In [32], singularities in the sliver states describing D-branes in the flat background was identified.[2] For the transverse directions, the sliver state satisfies

$$X\left(\frac{\pi}{2}\right)|\Psi\rangle = 0\,. \tag{7.18}$$

For the longitudinal directions the sliver state is invariant under the variation

$$\delta X(\sigma) = \begin{cases} \lambda & \text{for } 0 \le \sigma < \frac{\pi}{2}, \\ -\lambda & \text{for } \pi/2 < \sigma \le \pi. \end{cases} \tag{7.19}$$

In terms of the momentum, it is

$$(P_L - P_R)|\Psi\rangle = 0\,, \tag{7.20}$$

where

$$P_L \equiv \int_0^{\pi/2} P(\sigma)\,, \qquad P_R \equiv \int_{\pi/2}^{\pi} P(\sigma)\,. \tag{7.21}$$

Back to the pp-wave case, we will assume that these properties still hold for the pp-wave background and use them as our major guide to identify the D-brane projections. We also assume that the matter part of the string wave function corresponding to a Dp-brane extending along X_+, X_-, X_2,…, X_p directions is of the form

$$\Psi_p = \Phi(\vec{P}) \prod_{i=2}^{p} \psi_1(x_i, Y_i, Z_i) \prod_{i=p+1}^{D} \psi_2(x_i, Y_i, Z_i)\,, \tag{7.22}$$

and that each factor ψ_i $(i = 1, 2)$ factorized as

$$\psi_i = \Phi_i'(Y, Z)\Phi_i''(x)\,. \tag{7.23}$$

[1]The discrete spectrum is not fully explored yet, and it contains information about the thickness.

[2]It was shown that the singularities of the silver states are resolved when a constant B-field is turned on [33]. This is related to the noncommutative spacetime algebra for the zero mode as discussed in [34] and the second of [33].

Let us comment on each of the factors in (7.22). As we mentioned above, the ansatz only works for D-branes extended in the light cone directions. One can check that the projection (7.9) satisfies this condition (7.20). The factor $\Phi(\vec{P})$ is thus given by (7.9) and (7.12).

For the longitudinal directions, x should be allowed to end anywhere, so we should have $\Phi_1'' = 1$ $(x_1 \to -\infty, x_2 \to \infty)$ according to (7.15). Note that the coordinates Y, Z are both invariant under the variation (7.19), so the projector $\Phi_1'(Y, Z)$ written in terms of Y, Z, such as (7.13), automatically satisfies the desired criterion.

For the transverse directions, x is restricted to the place where the brane is localized. We take

$$\Phi_2''(x) = \Theta\left(x + \frac{b}{2}\right) - \Theta\left(x - \frac{b}{2}\right) \tag{7.24}$$

for a D-brane with finite thickness b. In terms of dimensionless quantities, the condition (7.18) should be replaced by

$$\frac{X(\pi/2)}{\sqrt{\alpha'}}|\psi_2\rangle = 0\,. \tag{7.25}$$

We will comment on this condition later.

The ratio of D-brane tensions

$$R := \frac{S_p}{S_{p+1}} = \frac{T_p V_p}{T_{p+1} V_{p+1}} = \frac{\langle\Psi_p|\Psi_p\rangle}{\langle\Psi_{p+1}|\Psi_{p+1}\rangle} \tag{7.26}$$

is then given by

$$R = \frac{\langle\psi_2|\psi_2\rangle}{\langle\psi_1|\psi_1\rangle} = \left.\frac{\langle\Phi_2''(x)|\Phi_2''(x)\rangle}{\langle\Phi_1''(x)|\Phi_1''(x)\rangle}\right|_{\vec{P}(\pi/2)=0}. \tag{7.27}$$

where we set $\vec{P}(\pi/2) = 0$ because of (7.12). The ratio R is thus

$$R = \left.\frac{b}{\int dx}\right|_{\vec{P}(\pi/2)=0} = \left.\frac{b}{\int \sqrt{\mu P_-(\pi/2)}dX(\pi/2)}\right|_{\vec{P}(\pi/2)=0} = \left.\frac{b}{\sqrt{\mu P_-(\pi/2)}L}\right|_{\vec{P}(\pi/2)=0}, \tag{7.28}$$

where L is the size of the transverse direction, and so it equals V_{p+1}/V_p. Setting $P_-(\pi/2) = 0$ in R, we find

$$\frac{T_p}{T_{p+1}} = \frac{S_p V_{p+1}}{S_{p+1} V_p} = \frac{b}{\sqrt{\mu P_-(\pi/2)}} = \infty \tag{7.29}$$

for any finite b. This is indeed what we expect since

$$\frac{T_p}{T_{p+1}} = 2\pi\sqrt{\alpha'} = \infty \tag{7.30}$$

in the tensionless limit.

In the zero tension limit the string length becomes infinity, it makes any finite thickness of the Dp-brane negligible so that the ratio (7.30) is not sensitive to the finite b value. But rigorously speaking, the transverse directions of the D-brane states are not well-defined, because the finiteness of b implies that the extent of $X(\pi/2)$ is $b/\sqrt{\mu P_-(\pi/2)} = \infty$ for $P_-(\pi/2) = 0$. To satisfy (7.25), we have to take $\alpha' \to \infty$ and $P_-(\pi/2) \to 0$ with $\alpha' P_-(\pi/2) \to \infty$. That is, a double scaling limit is involved. We expect this singularity to be renormalized when we include $1/\alpha'$ corrections to the calculation.

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8. Conclusion

In this paper, we studied and constructed the open bosonic string field theory in pp-wave background in the high energy limit. Following the recent work [16, 23], we showed that the Witten $*$-product for open string in the pp-wave background also admits a presentation in terms of the Moyal product. We find that the Moyal product has a universal noncommutativity of $\theta = 2$ common to all string modes in terms of the string bit basis.

We have also obtained the string field theory action in the tensionless limit. We plan to carry out a more detailed analysis of its dynamical aspects in the future. It would also be interesting to extend our results to the superstring case, which will be more relevant to the new pp/SYM duality [1].

The simplicity of the Moyal algebra makes it easy to find projective operators. We have proposed to identify some of these projectors with D-branes in the VSFT. The ratio of D-brane tensions can be easily calculated, but in the zero tension limit T_{p+1}/T_p is just zero. It will be useful if a perturbative method based on $\alpha'^{-1/2}$ expansion can be formulated. Then it may be possible to establish the more nontrivial result $T_{p+1}/T_p = 0 + (2\pi\alpha'^{1/2})^{-1} + \cdots$. A first step towards this goal is to identify the perturbation in (2.15). The perturbation will have to break the permutation symmetry S_∞.

We remark that this point of view of relating the finite tension string theory with the tensionless theory has the advantage that the the unperturbed tensionless theory has a much richer symmetry structure, as we demonstrated in this paper. This could be a convenient and powerful starting point for a background independent formulation of string theory. Note that the degrees of freedom (the string bits) employed in the tensionless theory look very different from the ordinary degrees of freedom in a string theory with finite tension. However this is perfectly acceptable. It may be useful to compare the situation with QCD. At zero coupling, the natural degrees of freedom are the free quarks and gluons. At nonzero coupling, the physical objects are colourless such as baryon. It may be more than an analogy to think of a finite tension string as a bound state of string bits.

A distinctive feature of the pp-wave background is that, as noted in [35], for weak couplings the tensionless string will not turn into a black hole as in the flat background, but will turn into giant gravitons. It would be interesting to see this more explicitly in the SFT. One may also wonder whether there is a description of the theory in which the giant gravitons are the fundamental objects. On the other hand, the commutative coordinate in the 3-string vertex we constructed is convenient for the Dirichlet boundary condition for the transverse coordinates. This may suggest another description in terms of D-instantons in the zero tension limit of the pp-wave SFT.

Very recently the light cone superstring field theory in pp-wave background is considered in [36]. Witten's open string field theory is covariant and the interaction of strings is characterized by the midpoint. This is in contrast with the situation in the light cone string field theory where interaction is described in terms of the splitting and jointing of strings. Another major difference is that the product in light cone string field theory cannot be presented as the Moyal product. Nevertheless it would be interesting to compare these results in the high energy limit.

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It has been conjectured [37] that in the high energy limit string theory in the AdS background has infinite higher spin gauge symmetries. Their holographic duals are higher spin conserved currents in the Yang-Mills theory. Such a gauge theory in AdS has already been constructed in [38]. It would be interesting to see whether it corresponds to the string theory. Incidentally, in another limit with $N \to \infty$ and $g^2 N$ fixed, infinite higher spin gauge symmetries are also expected in AdS space [39]. See also [40]. In both cases, something like the Higgs mechanism should occur to break the higher spin gauge symmetries when the coupling and energies are finite. We hope that this work will help understanding these important issues about the short-distance physics of string theory.

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